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HYDRODYNAMICS IN TECHNOLOGICAL PROCESSES

THE BUSEMANN FLOW FOR A ONE-VELOCITY MODEL OF A HETEROGENEOUS MEDIUM

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A solution of the self-similar problem of a one-velocity multicomponent flow of a heterogeneous medium near a cone with an attached shock wave (an analog of the Busemann problem for a perfect gas) which accounts for the internal forces of interfractional interaction has been obtained. In calculating a conical shock wave, a shock adiabat of a mixture coordinated with the equation of the one-velocity model was used.

There are two approaches to derivation of the equations of the one-velocity model of a heterogeneous medium. In the first, the equations of the model follow from the equations of a multivelocity continuum [1], in which one should take the velocities of all the constituting fractions as equal. With such an approach, the forces of interfractional interaction $\mathbf{f}_{ij}$ are equal to zero, since they are proportional to the difference of the velocities of the fractions indicated by the subscripts i and j. The model obtained in this way was considered in [2], where it is also shown that the Cauchy problem for the equations used is correct only in a limited range of velocities. In particular, in a flow of a vapor-gas dropping mixture the Cauchy problem is correct at a velocity which does not exceed 55 m/sec in absolute value.

In the present work, the one-velocity model of a multicomponent medium is constructed on another principle. In it, the coincidence of the velocities of the fractions is assumed from the onset. In order to ensure equal accelerations for each of the inhomogeneous fractions of the mixture, internal interfractional forces differing from zero are introduced into the equations of motion. We note that for the model of a medium in which the indicated forces are taken into account the Cauchy problem is correct with any number of fractions in the mixture and at an arbitrary flow velocity.

For a model of a medium without internal interfractional forces, a solution is given in [3] for the problem of mixture flow near a cone. The solution was obtained due to the presence of an analytical relation for the mixture isoentropic curve, which was used in representing the Bernoulli integral. For the variant of the model of a medium with nonzero interfractional forces one has failed to solve the Busemann problem as was done in [3] for the absence of an analytical expression for the isoentropic curve. Nevertheless, a self-similar solution of the problem exists. In the present work, a self-similar solution of the Busemann problem is presented for a model of a mixture in which the internal forces of interfractional interaction are taken into account. In constructing the solution, the shock adiabat of a mixture from [4] was used, which is coordinated with the one-velocity model equations.

Equations of the Model. We will consider a one-velocity, n-component mixture. The differential equations that express the laws of mass, momentum, and energy conservation for the ith compressible component have the form

$$\frac{\partial \alpha_i \rho_i^0}{\partial t} + \text{div}\,(\alpha_i \rho_i^0 \mathbf{u}) = \sum_{j=1}^{n(j\neq i)} J_{ij}\,, \tag{1}$$

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$$\rho_i \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{u}\right] + \mathrm{grad}\,(\alpha_i \partial) + \sum_{j=1}^{n(j\neq i)} J_{ij}\mathbf{u} = \alpha_i \mathbf{F} + \sum_{j=1}^{n(j\neq i)} \mathbf{f}_{ij}\,, \tag{2}$$

$$\frac{\partial}{\partial t}\left[\alpha_i \rho_i^0 \left(\varepsilon_i + \frac{1}{2}\,|\mathbf{u}|^2\right)\right] + \mathrm{div}\left[\alpha_i \rho_i^0 \left(\varepsilon_i + \frac{1}{2}\,|\mathbf{u}|^2\right)\mathbf{u}\right] + \mathrm{div}\,(\alpha_i p\mathbf{u}) + \mathrm{div}\,(\alpha_i \mathbf{W}_i) =$$

$$= \alpha_i \mathbf{F}\cdot\mathbf{u} + \sum_{j=1}^{n(j\neq i)} \left(\mathbf{f}_{ij}\cdot\mathbf{u} + R_{ij} + Q_{ij}\right). \tag{3}$$

We assume that the heat transfer by the Fourier law $\mathbf{W}_i = -\chi_i\ \mathrm{grad}\ T_i$ occurs in three components that form the "skeleton" of the medium.

In order to eliminate the densities of forces $\mathbf{f}_{ij}$ from Eq. (3), we form the scalar product of Eq. (2) with the velocity vector $\mathbf{u}$ and subtract it from Eq. (3). As a result, we obtain the heat-inflow equation:

$$\rho_i \left(\frac{\partial \varepsilon_i}{\partial t} + (\mathbf{u}\cdot\nabla)\,\varepsilon_i\right) + \frac{\alpha_i p}{\rho_i}\left[\sum_{j=1}^{n(j\neq i)} J_{ij} - \left(\frac{\partial \rho_i}{\partial t} + (\mathbf{u}\cdot\nabla)\,\rho_i\right)\right] + \mathrm{div}\,(\alpha_i \mathbf{W}) = \sum_{j=1}^{n(j\neq i)} (R_{ij} + Q_{ij}) - \left(\varepsilon_i - \frac{1}{2}\,|\mathbf{u}|^2\right)\sum_{j=1}^{n(j\neq i)} J_{ij}\,, \tag{4}$$

which does not contain the densities of the forces of interfractional interaction.

Summation of each of Eqs. (1)–(3) over the subscript i from 1 to n yields the mass, momentum, and energy conservation laws for the mixture as a whole:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\,(\rho\mathbf{u}) = 0\,, \quad \rho\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{u}\right] + \mathrm{grad}\,p = \mathbf{F}\,,$$

$$\frac{\partial}{\partial t}\left[\rho\left(\varepsilon + \frac{1}{2}\,|\mathbf{u}|^2\right)\right] + \mathrm{div}\left[\rho\left(\varepsilon + \frac{1}{2}\,|\mathbf{u}|^2 + \frac{p}{\rho}\right)\mathbf{u} + \sum_{i=1}^{n} \alpha_i \mathbf{W}_i\right] = \mathbf{F}\cdot\mathbf{u}\,, \tag{5}$$

where

$$\varepsilon = \frac{1}{\rho}\sum_{i=1}^{n} \rho_i \varepsilon_i\,. \tag{6}$$

In derivation of system (5) the following equalities were used:

$$Q_{ij} = -\,Q_{ji}\,,\quad R_{ij} = -\,R_{ji}\,,\quad J_{ij} = -\,J_{ji}\,,\quad \mathbf{f}_{ij} = -\,\mathbf{f}_{ji}\,. \tag{7}$$

By ignoring the heat-transfer processes and we may consider the equations of system (5) as a "gas-dynamical" core of the one-velocity model of a heterogeneous medium because they coincide fully with the equations of the single-phase gas dynamics.

We will consider a mixture flow, in which we neglect the influence of heat-transfer processes and phase (chemical) transitions, as well as the action of mass forces. The equation of the first law of thermodynamics has the form

$$T_i\left[\frac{\partial S_i}{\partial t} + (\mathbf{u}\cdot\nabla)\,S_i\right] = \frac{\partial \varepsilon_i}{\partial t} + (\mathbf{u}\cdot\nabla)\,\varepsilon_i + \alpha_i p\left[\frac{\partial}{\partial t}\left(\frac{1}{\rho_i}\right) + (\mathbf{u}\cdot\nabla)\left(\frac{1}{\rho_i}\right)\right] + \sum_{j=1}^{n(j\neq i)} \mathbf{f}_{ij}\cdot\mathbf{u}\,. \tag{8}$$

Here S_i and T_i are the entropy and temperature of the ith component of the mixture. On the other hand, the heat inflow equation (4) for the ith fraction of the mixture can be written as

$$\frac{d\varepsilon_i}{dt} + \alpha_i p \frac{d}{dt}\left(\frac{1}{\rho_i}\right) = 0 , \tag{9}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{u}\cdot\nabla)$. Relation (8), subject to (9), takes the form

$$T_i\left[\frac{\partial S_i}{\partial t} + (\mathbf{u}\cdot\nabla)\, S_i\right] = \sum_{j=1}^{n(j\neq i)} \mathbf{f}_{ij}\cdot\mathbf{u} . \tag{10}$$

If we sum up Eq. (10) over i from 1 to n, we obtain the expression

$$\sum_{i=1}^{n} T_i\left[\frac{\partial S_i}{\partial t} + (\mathbf{u}\cdot\nabla)\, S_i\right] = T\left[\frac{\partial S}{\partial t} + (\mathbf{u}\cdot\nabla)\, S\right] = 0 , \tag{11}$$

where T is the average temperature of the mixture; $S = \sum_{i=1}^{n} S_i$ is the specific entropy of the mixture as a whole. From Eq. (11) it is seen that the entropy of the mixture along the trajectory of motion of a mixture particle remains unchanged in contrast to the entropies of the mixture components (10). The latter will also be constant if we neglect the work of the forces of interfractional interaction. This makes it possible to infer, in particular, that within the framework of the model of the medium in which the forces of interfractional interaction are taken into account, it is impossible to obtain an expression for the mixture isoentropic curve in the way it was done in [2].

Henceforth, in describing the behavior of the mixture components, we will use, for the sake of definiteness, the equations of state of the form

$$\varepsilon_i = \frac{p - c_{*i}^2\,(\rho_i^0 - \rho_{*i})}{\rho_i^0\,(\gamma_i - 1)} . \tag{12}$$

For an n-component mixture with the first m compressible fractions each of which obeys the state equations (12), Eq. (6) takes the form

$$\varepsilon = \frac{1}{\rho}\left[b_m + pB_m + \sum_{i=1}^{m-1} \alpha_i\,(b_{im} - a_{im}\rho_i^0 + pB_{im}) + \sum_{j=m+1}^{n} \alpha_j \rho_j^0 \varepsilon_j\right] - a_m , \tag{13}$$

where the following coefficients are introduced: $a_i = c_{*i}^2 B_i$; $a_{im} = a_i - a_m$; $b_i = a_i\rho_{*i}$; $b_{im} = b_i - b_m$; $B_i = 1/(\gamma_i - 1)$; and $B_{im} = B_i - B_m$. The equation of state (12) for the ith fraction can be rewritten in the form

$$\varepsilon_i = \frac{b_i + pB_i}{\rho_i^0} - a_i . \tag{14}$$

For the adiabatic variant of the model to be considered in what follows, the full system of equations of the n-component mixture with the first m compressible fractions includes: a system of differential equations (5) for the mixture as a whole; Eqs. (1) and (4) for the compressible fractions in which the subscript i takes the values 1, ..., $m - 1$, as well as Eqs. (1) for the incompressible fractions with subscripts $i = m + 1$, ..., n. If we eliminate the variables ε and ε_i from this system, employing relations (13) and (14) for this purpose, we will obtain a closed system of $n + m + d$ (d

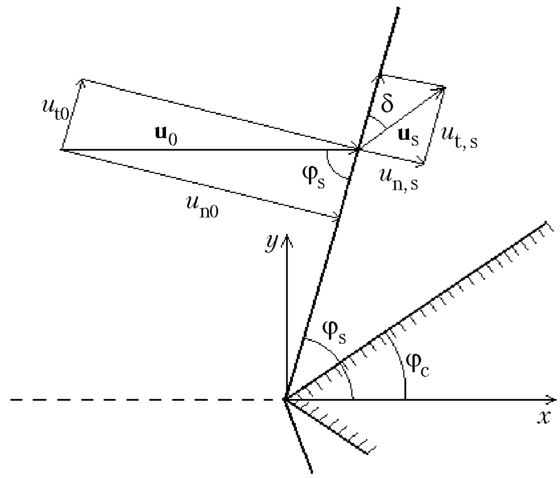


Fig. 1. Flow of a mixture near a cone with an attached shock wave.

is the dimensionality of the problem) quasilinear first-order partial differential equations for determining the same number of unknowns, which for completeness' sake should be supplemented with initial and boundary conditions.

Flow Near a Cone. We will consider a stationary multicomponent mixture flow at a zero angle of attack around a cone with a semivertex angle φ_c for a regime with an attached conical shock wave (Fig. 1). Expanding the velocity vectors before and after the shock into normal ones and those tangent to the attached shock-wave front, we obtain

$$u_{n0} = |\mathbf{u}_0| \sin \varphi_s , \quad u_{t0} = |\mathbf{u}_0| \cos \varphi_s , \quad u_{n,s} = |\mathbf{u}_s| \sin \delta , \quad u_{t,s} = |\mathbf{u}_s| \cos \delta .$$

Since the tangential velocity components are identical for both vectors $\mathbf{u}_0$ and $\mathbf{u}_s$, we have

$$|\mathbf{u}_0| \cos \varphi_s = |\mathbf{u}_s| \cos \delta . \tag{15}$$

The velocity vector $\mathbf{u}_s$, unlike the plane case (flow around a wedge), is noncollinear with the cone surface generatrix. In transition through the attached shock wave, in the Rankine–Hugoniot relations for a normal shock one should use normal velocity components:

$$\rho_0 |\mathbf{u}_0| \sin \varphi_s = \rho_s |\mathbf{u}_s| \sin \delta , \quad p_0 + \rho_0 |\mathbf{u}_0|^2 \sin^2 \varphi_s = p_s + \rho_s |\mathbf{u}_s|^2 \sin^2 \delta . \tag{16}$$

Relations (15) and (16) are considered together with the shock adiabat of the n-component mixture [4] in which the first m fractions are assumed compressible:

$$\frac{1}{\rho_s}\left[b_m + p_s B_m + \sum_{i=1}^{m-1} \alpha_{is} (b_{im} - a_{im}\rho_{is}^0 + p_s B_{im}) + \sum_{j=m+1}^{n} \alpha_{js}\rho_j^0 \varepsilon_j \right] -$$

$$- \frac{1}{\rho_0}\left[b_m + p_0 B_m + \sum_{i=1}^{m-1} \alpha_{i0} (b_{im} - a_{im}\rho_{i0}^0 + p_0 B_{im}) + \sum_{j=m+1}^{n} \alpha_{j0}\rho_j^0 \varepsilon_j \right] = \frac{p_s + p_0}{2}\left(\frac{1}{\rho_0} - \frac{1}{\rho_s} \right),$$

$$\rho_{is}^0 = \frac{b_i + p_s B_i - \dfrac{p_s |\mathbf{u}_s| (\rho_s - \rho_0)}{\rho_0 (|\mathbf{u}_s| - |\mathbf{u}_0|)}}{\dfrac{|\mathbf{u}_0|^2}{2} - \dfrac{|\mathbf{u}_s|^2}{2} + \dfrac{1}{\rho_{i0}^0}\left[b_i + p_0 B_i - \dfrac{p_0 |\mathbf{u}_0| (\rho_s - \rho_0)}{\rho_s (|\mathbf{u}_s| - |\mathbf{u}_0|)} \right]} , \quad \alpha_{is} = \alpha_{i0} \frac{\rho_s \rho_{i0}^0}{\rho_0 \rho_{is}^0} , \quad i = 1, ..., m-1 , \tag{17}$$

$$\alpha_{js} = \alpha_{j0}\,\frac{\rho_s}{\rho_0}\,, \quad j = m+1, ..., n\,.$$

In order to find the distribution of parameters between the attached shock wave and the cone surface it is necessary to integrate the system of differential equations of the model which describes the stationary axisymmetrical adiabatic mixture flow. This system of equations follows from the general equations of the model given in the previous section and has the form

$$\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + u\,\frac{\partial \rho}{\partial x} + v\,\frac{\partial \rho}{\partial y} = -\,\frac{v\rho}{y}\,, \quad u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} + \frac{1}{\rho}\,\frac{\partial p}{\partial x} = 0\,, \quad u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} + \frac{1}{\rho}\,\frac{\partial p}{\partial y} = 0\,,$$

$$A_1\left(u\,\frac{\partial \rho}{\partial x} + v\,\frac{\partial \rho}{\partial y}\right) + A_2\left(u\,\frac{\partial p}{\partial x} + v\,\frac{\partial p}{\partial y}\right) +$$

$$+\sum_{i=1}^{m-1}\left[A_{i+2}\left(u\,\frac{\partial \rho_i^0}{\partial x} + v\,\frac{\partial \rho_i^0}{\partial y}\right) + A_{i+m+1}\left(u\,\frac{\partial \alpha_i}{\partial x} + v\,\frac{\partial \alpha_i}{\partial y}\right)\right] + \sum_{j=m+1}^{n} A_{j+m}\left(u\,\frac{\partial \alpha_j}{\partial x} + v\,\frac{\partial \alpha_j}{\partial y}\right) = 0\,,$$

$$-\frac{1}{\rho}\left(u\,\frac{\partial \rho}{\partial x} + v\,\frac{\partial \rho}{\partial y}\right) + \frac{1}{\rho_i^0}\left(u\,\frac{\partial \rho_i^0}{\partial x} + v\,\frac{\partial \rho_i^0}{\partial y}\right) + \frac{1}{\alpha_i}\left(u\,\frac{\partial \alpha_i}{\partial x} + v\,\frac{\partial \alpha_i}{\partial y}\right) = 0\,, \tag{18}$$

$$\frac{B_i}{p}\left(u\,\frac{\partial p}{\partial x} + v\,\frac{\partial p}{\partial y}\right) - \frac{b_i + p\,(1 + B_i)}{p\rho_i^0}\left(u\,\frac{\partial \rho_i^0}{\partial x} + v\,\frac{\partial \rho_i^0}{\partial y}\right) - \frac{1}{\alpha_i}\left(u\,\frac{\partial \alpha_i}{\partial x} + v\,\frac{\partial \alpha_i}{\partial y}\right) = 0\,, \quad i = 1, ..., m-1\,,$$

$$-\frac{1}{\rho}\left(u\,\frac{\partial \rho}{\partial x} + v\,\frac{\partial \rho}{\partial y}\right) + \frac{1}{\alpha_j}\left(u\,\frac{\partial \alpha_j}{\partial x} + v\,\frac{\partial \alpha_j}{\partial y}\right) = 0\,, \quad j = m+1, ..., n\,.$$

Here, we put

$$A_1 = -\frac{1}{\rho}\left[b_m + p\,(1 + B_m) + \sum_{i=1}^{m-1}\alpha_i\,(b_{im} - \rho_i^0 a_{im} + pB_{im}) + \sum_{j=m+1}^{n}\alpha_j A_{j+m}\right],$$

$$A_2 = B_m + \sum_{i=1}^{m-1}\alpha_i B_{im}\,, \quad A_{i+2} = -\,\alpha_i a_{im}\,, \quad A_{i+m+1} = b_{im} - \rho_i^0 a_{im} + pB_{im}\,, \quad A_{j+m} = \rho_j^0\varepsilon_j = \text{const}\,.$$

We will seek the self-similar solution of the problem, i.e., the solution in which all dependent variables are functions of only one independent variable, $\xi = y/x$. Subject to the relations $\frac{\partial}{\partial x} = -\frac{\xi d}{x d\xi}$ and $\frac{\partial}{\partial y} = \frac{1 d}{x d\xi}$, system (18) is reduced to the following ordinary differential equations:

$$\rho\left(\frac{dv}{d\xi} - \xi\,\frac{du}{d\xi}\right) + (v - u\xi)\,\frac{d\rho}{d\xi} = -\,\frac{v\rho}{\xi}\,, \quad (v - u\xi)\,\frac{du}{d\xi} - \frac{\xi}{\rho}\,\frac{dp}{d\xi} = 0\,, \quad (v - u\xi)\,\frac{dv}{d\xi} + \frac{1}{\rho}\,\frac{dp}{d\xi} = 0\,, \tag{19}$$

$$A_1\,\frac{d\rho}{d\xi} + A_2\,\frac{dp}{d\xi} + \sum_{i=1}^{m-1}\left(A_{i+2}\,\frac{d\rho_i^0}{d\xi} + A_{i+m+1}\,\frac{d\alpha_i}{d\xi}\right) + \sum_{j=m+1}^{n} A_{j+m}\,\frac{d\alpha_j}{d\xi} = 0\,, \tag{20}$$

$$-\frac{1}{\rho}\frac{d\rho}{d\xi}+\frac{1}{\rho_i^0}\frac{d\rho_i^0}{d\xi}+\frac{1}{\alpha_i}\frac{d\alpha_i}{d\xi}=0\,, \tag{21}$$

$$\frac{B_i}{p}\frac{dp}{d\xi}-\frac{b_i+p\,(1+B_i)}{p\rho_i^0}\frac{d\rho_i^0}{d\xi}-\frac{1}{\alpha_i}\frac{d\alpha_i}{d\xi}=0\,,\quad i=1,\,...,\,m-1\,, \tag{22}$$

$$-\frac{1}{\rho}\frac{d\rho}{d\xi}+\frac{1}{\alpha_j}\frac{d\alpha_j}{d\xi}=0\,,\quad j=m+1,\,...,\,n\,. \tag{23}$$

Equations (21) and (23) can be integrated to give

$$\alpha_i=\alpha_{is}\frac{\rho}{\rho_s}\frac{\rho_{is}^0}{\rho_i^0}\,, \tag{24}$$

$$\alpha_j=\alpha_{js}\frac{\rho}{\rho_s}\,. \tag{25}$$

By substituting the resulting equations (24) and (25), as well as the relation

$$\frac{d\rho_i^0}{d\xi}=\frac{p\rho_i^0}{b_i+pB_i}\left(\frac{B_i}{p}\frac{dp}{d\xi}-\frac{1}{\rho}\frac{d\rho}{d\xi}\right),$$

which follows from Eqs. (21) and (22), into Eq. (20), we obtain the equation

$$D_1\frac{dp}{d\xi}+D_2\frac{d\rho}{d\xi}=0\,, \tag{26}$$

where

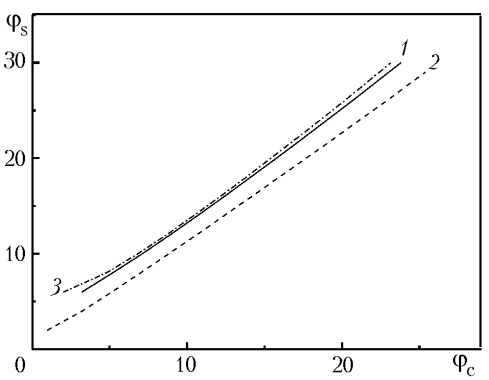
$$D_1=B_m-\sum_{i=1}^{m-1}\frac{\alpha_{is}\rho\rho_{is}^0\,(b_iB_m-b_mB_i)}{\rho_s\rho_i^0\,(b_i+pB_i)}\,;\quad D_2=-\frac{1}{\rho}\left\{b_m+p\left[1+B_m-\sum_{i=1}^{m-1}\frac{\alpha_{is}\rho\rho_{is}^0\,(b_{im}+pB_{im})}{\rho_s\rho_i^0\,(b_i+pB_i)}\right]\right\}.$$

We will rewrite the system of differential equations (19), (22), and (26) in a form convenient for integration:

$$\frac{dp}{d\xi}=\frac{\rho v\,(v-u\xi)\,D_2}{(v-u\xi)^2\,D_1+(1+\xi^2)\,D_2}\,,\quad \frac{du}{d\xi}=\frac{v\xi D_2}{(v-u\xi)^2\,D_1+(1+\xi^2)\,D_2}\,,$$

$$\frac{dv}{d\xi}=-\frac{vD_2}{(v-u\xi)^2\,D_1+(1+\xi^2)\,D_2}\,,\quad \frac{d\rho}{d\xi}=-\frac{\rho v\,(v-u\xi)\,D_1}{(v-u\xi)^2\,D_1+(1+\xi^2)\,D_2}\,, \tag{27}$$

$$\frac{d\rho_i^0}{d\xi}=\frac{\rho_i^0 v\,(v-u\xi)\,(pD_1+\rho B_iD_2)}{(b_i+pB_i)\,[(v-u\xi)^2\,D_1+(1+\xi^2)\,D_2]}\,,\quad i=1,\,...,\,m-1\,.$$



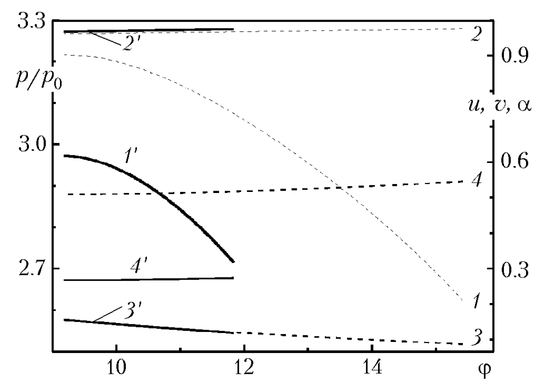


Fig. 2. Dependence of φ_s on φ_c: 1, 3) with an incompressible liquid fraction; 2) with allowance for its compressibility.

Fig. 3. The flow parameters vs. the angle φ: 1, 1′) p/p_0; 2, 2′); u; 3, 3′), v; 4, 4′), α. Curves 1′–4′ were obtained with allowance for the compressibility of the liquid fraction.

The number of equations in system (27) is proportional to the number of compressible fractions in the mixture. After system (27) has been integrated, the remaining parameters of flow, namely, the volumetric fractions of compressible and incompressible components, α_i and α_j, of the mixture are determined from relations (24) and (25). We note that if, among the compressible fractions, there are ideal gases for which the coefficients $b_i = 0$, then for them, after Eqs. (22) have been integrated, we can obtain the dependences

$$\rho_i^0 = \rho_{is}^0 \frac{p}{p_s} \left(\frac{\rho_s}{\rho} \right)^{1/B_i},$$

thus reducing the number of differential equations in system (27).

The solution of the system of equations (27) must, first, satisfy the boundary condition of zero leakage through the surface of the cone (at $\varphi = \varphi_c$). Second, at the shock-wave front, relations (15)–(17), which involve the inclination angle of the shock wave $\varphi_s = \arctan(\xi_s)$, should hold. It should be kept in mind that this angle is unknown a priori; therefore, we will replace the boundary-value problem with complex nonlinear conditions on the boundaries of the domain by the Cauchy problem, thus substantially simplifying the algorithm of calculation. For this purpose, we arbitrarily prescribe the value of ξ_s; then from relations (15)–(17) we will determine the flow parameters at the shock-wave front. Thereafter we integrate system (27) numerically from $\xi = \xi_s$ up to such a value of ξ at which the condition of no-leakage through the cone surface is satisfied:

$$\xi_c = \frac{v(\xi_c)}{u(\xi_c)}. \tag{28}$$

When condition (28) is satisfied, the direction of flow motion is parallel to the cone surface generatrix. This will yield the semivertex angle of the cone in a flow: $\varphi_c = \arctan(\xi_c)$. By varying ξ_s we will find the function $\varphi_c(\varphi_s)$, the inversion of which will yield the sought-for function $\varphi_s(\varphi_c)$.

As an example of the use of the above-given method of solving the Busemann problem, we calculated flow of a gas-liquid mixture with an incompressible liquid component ($\gamma = 1.4$, $\rho_{10}^0 = 1.19$ kg/m^3, $\rho_{20}^0 = 1000$ kg/m^3) near the cone with velocity $u_0 = 295.1$ m/sec (M = 10), initial gas content in the mixture $\alpha = 0.8$ and pressure $p_0 = 10^5$ Pa, and the function $\varphi_s(\varphi_c)$ (curve 1 in Fig. 2) was found. The same figure contains the corresponding dependence obtained at the same initial data but with account for the compressibility of the water component of the mixture (curve 2, $\gamma_2 = 5.59$, $\rho_{*2} = 1000$ kg/m^3, $c_{*2} = 1500$ m/sec). Curve 3 was calculated with the use of the model of [2] by the algorithm from [3] for a gas-liquid mixture with an incompressible liquid component. The corresponding systems of differential equations were integrated numerically using a fourth-order Runge–Kutta method.

The distribution of the flow parameters downstream of the shock wave — the relative pressure, projections of velocities u and v onto the coordinate axes x and y, as well as the gas content α as a function of the angle φ for the flow version with $u_0 = 150$ m/sec, initial gas concentration in the mixture $\alpha_0 = 0.7$ for a cone with an angle $\varphi_c = 9.18^o$ (for a mixture with an incompressible liquid fraction) — presented in Fig. 3. The corresponding dependences obtained with allowance for the liquid compressibility are presented in Fig. 3 by curves 1′–4′. From the data presented in Fig. 3, the dependence of the results of computations on the elastic properties of the weakly compressible liquid fraction is seen.

NOTATION

c_{*i}, a constant in the equation of state, m/sec; $\mathbf{f}_{ij}$, density of the internal force of interfractional interaction between the ith and jth mixture components, kg/(m^2·sec^2); $\mathbf{F}$, density of the mass force, kg/(m^2·sec^2); J_{ij}, intensity of the conversion of mass from the ith fraction into the jth one in a unit volume of mixture, kg/(m^3·sec); M, Mach number; m, number of compressible fractions in a mixture; n, total number of fractions in a mixture; p, pressure, Pa; Q_{ij}, heat release per unit time per unit volume of mixture as a result of conversion of the fraction i into the fraction j, kg/(m·sec^3); R_{ij}, quantity of heat per unit time per unit volume of the mixture transmitted from the jth fraction to the ith one due to radiative heat transfer; t, time, sec; T, temperature, K; $\mathbf{u}$, velocity vector, u and v, its projections onto the coordinate axes x and y, m/sec; $\mathbf{W}$, heat-flux density vector, kg/sec^3; α_i, volumetric fraction of the ith component of the mixture; γ_i, a constant in the equation of state; δ, angle between the attached shock wave and velocity vector downstream of its front, deg; ε, specific internal energy, m^2/sec^2; $\xi = y/x$, self-similar variable; ρ, density of the mixture, kg/m^3; ρ_i^0, true density of the ith fraction, kg/m^3; ρ_i, reduced density of the ith component, kg/m^3; ρ_{*i}, a constant in the equation of state, kg/m^3; φ, angle, deg; φ_c, semivertex angle of the cone, deg; φ_s, angle of inclination of the attached shock wave, deg; χ, thermal conductivity, kg·m/(sec^3·K); $\nabla = (\partial/\partial x, \partial/\partial y)$. Subscripts and superscripts: 0, in the incoming (nonperturbed) flow; c, on the cone surface; n and t, normal and tangential components; s, at the shock front.

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